CCSSM:

Teaching in Grades 3 and

How is each Common Core State Standard for Mathematics different from each old objective?

By Angela T. Barlow and Shannon Harmon

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To support mathematics educators as they consider the implications of the Common Core State Standards for Mathematics (CCSSM) on instruction and assessment, *Teaching Children Mathematics* is publishing a series of articles. In this third feature of the series, authors Barlow and Harmon suggest implementation strategies for grades 3 and 4. The next article covers additional topics, ideas, and commentary addressing grades 5 and 6.

Mrs. Hernandez teaches third grade. The objective in her old curriculum read, "Identify fractions as equal-size parts of a whole." As she looked at the Common Core State Standards for Mathematics (CCSSI 2010), she read the following:

> 3.NF.1 – Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

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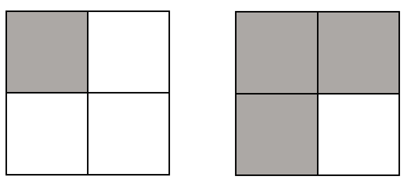
Mrs. Hernandez thought, "That's just a fancy way of saying the same thing." But is it? If Mrs. Hernandez's instruction supported students in meeting the previous objective, will it suffice for meeting this Standard from CCSSM?

At a time when many of us are looking for commonalities between our current curriculum framework and CCSSM, the purpose of this article is to help third- and fourth-grade teachers think about the *differences*. How is each Standard different from each old objective? How can I better understand CCSSM? How do my instructional decisions relate to the Standards for Mathematical Practice (CCSSI 2010)? In answering these questions, we have elected to use Standard 3.NF.1, although the processes we describe can be applied to any third- and fourth-grade Standards. Through this example, we aim to support you in better understanding CCSSM as well as provide you with a means for navigating the document.

FIGURE 1

CCSSM Standard 3.NF.1 expects students to identify one-fourth as a quantity that can be counted and formalize this knowledge by attaching fraction symbols to pictures and words.

(a) (b)



Using the CCSSM document

Some Standards may be expressed in unfamiliar language, and when reading a Standard, you are appraising "what students should understand and be able to do" (CCSSI 2010, p. 5). The Standards identify an end goal of a unit of instruction that encompasses more than a skill that may be taught in one or two lessons. Therefore, the first step in understanding the document is to "unpack" the Standard, that is, identify the mathematics contained within it. For this, we draw heavily on Wiggins and McTighe's *Understanding by Design* (*UBD*) (2005). According to *UBD*, unpacking a Standard requires identifying the understanding, knowledge, and skills associated with it. The best way to illustrate our understanding of these ideas is to return to the Standard above and "unpack" it.

Unpacking a Standard—Prerequisite knowledge

Before unpacking the Standard, one must think about the prerequisite knowledge that students should have. In terms of CCSSM, this means examining Standards from earlier grade levels to identify those that provide a foundation for the one you are unpacking. For Standard 3.NF.1, we reviewed the second-grade Standards:

> 2.G.3 – Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

Based on the second-grade Standard, students should enter third grade knowing the following for halves, thirds, and fourths—

- **how** to partition a shape into equal-size parts;
- **how** to describe those parts using the language of fractions; and
- **that** the parts collectively represent the whole.

Having identified the prerequisite knowledge, we are ready to unpack the third-grade Standard.

Unpacking a Standard—Understanding

To identify understanding, we consider the following sentence: As a result of this unit of instruction, students will understand that.... To complete it, we ask ourselves, What are the big mathematical ideas associated with this Standard? For Standard 3.NF.1, we obviously want our students to understand that a fraction is a part of a whole, but what else is embedded in the Standard? If we dissect the wording, students must move beyond a simple counting of "parts shaded" and "parts that make up the whole." Instead, they begin with unit fractions (what CCSSM terms *fractions of the form 1/b*) and then move toward counting the number of unit fractions.

To clarify these ideas, consider the representations in **figure 1**. One way to think about these representations is to count how many parts are shaded (one in **fig. 1a** and three in **fig. 1b**) to represent your fraction numerator. Then count how many parts are in the whole (four in both figures), and let this be your fraction denominator. But this line of thinking does not really address the mathematics contained in Standard 3.NF.1. Instead, the Standard's expectation is for students to identify the one-fourth in **figure 1a** and understand it as a quantity that can be counted. Then, in **figure 1b**, students are to count that there are three of these quantities, that is, three of these one-fourth parts, which is three-fourths. Recognizing the prerequisite knowledge, however, students will have some background in identifying and counting these one-fourth parts. With this Standard, they formalize this knowledge by attaching fraction symbols to their pictures and words. Therefore, our unpacking of this Standard revealed that students will understand that—

- **a fraction** symbol represents a quantity that involves parts of a whole; and
- **multiple** parts of a whole can be counted and represented by a fraction symbol.

CCSSM in the Classroom

The following vignette briefly describes the implementation of the Pizza task in a third-grade classroom.

Teacher: Let's all read the problem in our heads while Jeremy reads the problem aloud. [*Jeremy reads the problem*.] Now, what are some things we know about the problem? Jonna?

Jonna: Three kids are sharing a pizza.

Teacher: Thank you, Jonna. What's something else we know? Ray?

Ray: They all get the same amount.

Teacher: Nice. What else do we know?

Todd: It's a large pizza.

Teacher: Thank you, Todd. Would someone like to share any other ideas?

Amy: The pizza is square, which is weird because pizzas are supposed to be round!

Teacher: That's interesting, Amy. I usually eat round pizzas, too. But for this problem, we are working with a square pizza. Can someone tell us what the problem is asking us to do?

Job: We have to draw on the pizza.

Teacher: Would someone like to add to what Job said? Angelyn?

Angelyn: We have to draw how much Elena gets.

Teacher: Nice. So we will start by working on the problem for about five minutes, and then we will see how far we have gotten. [*Students work on the problem for the next few minutes. As they work, the teacher circulates the room, monitoring their progress and using questions to direct or redirect their thoughts as necessary. When time is called, she selects three students to share their work with the class. As they share their solutions, the teacher asks questions and guides the classroom discussion. A sampling of these questions follows.*]

Q1: Amy, will you explain to us why you decided to divide the pizza like that?

Q2: Do you all agree with Amy's reasoning? Jonna, why do you agree?

Q3: Randy, how do you know your picture is right?

Q4: How are Amy's and Randy's pictures alike, Todd?

Q5: Talk with your partner about what Job might do to fix his picture.

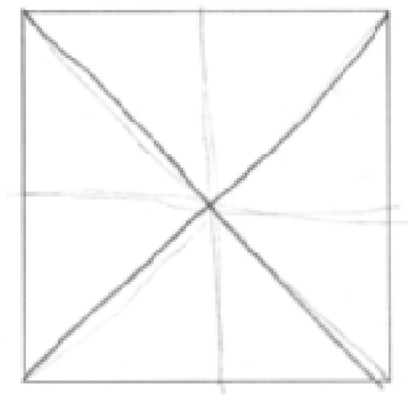
Q6: As you look at these different representations, what are some things you are noticing? Think about that for a moment. Now talk with your partner.

FIGURE 2

Incorrect responses—like Carlos's and many of his classmates', who partitioned the square into fourths or eighths but not into thirds—reveal when students do not possess the prerequisite knowledge needed for the unit.

Luis, Elena, and Leslie plan to share 1 large, square pizza. Each person will get an equal amount.

1. Show on the picture how much pizza Elena will get.



2. What fraction of the pizza will Elena get?

$\frac{1}{4}$

Unpacking a Standard—Knowledge

To identify the knowledge pieces of a Standard, including terms, properties, or other factual components, complete the next sentence. For Standard 3.NF.1, students will know the following:

- **A fraction** symbol like $1/b$ is one part of a whole and the whole is partitioned into b equal parts.
- **A fraction** a/b is the quantity formed by a parts of size $1/b$.
- **The numerator** of a fraction is the "top number," and it tells how many parts of a certain size are being counted.
- **The denominator** of a fraction is the "bottom number" and tells the number of equal parts the whole is partitioned into.

Unpacking a Standard—Skills

Finally, unpacking the Standard includes identifying the skills that students must gain to meet the Standard's expectations. We use this sentence starter: For Standard 3.NF.1, students will be able to do the following:

- **Write** the fraction symbol for a given representation of a fraction of the form 1/b.
- **Write** the fraction symbol for a given representation of a fraction of the form a/b.
- **Represent** a given fraction of the form $1/b$ with pictures and/or manipulatives.
- **Represent** a given fraction of the form a/b with pictures and/or manipulatives.
- **Count** the parts of a whole using the language of fractions such as one-fourth, two-fourths, three-fourths, four-fourths.
- **Given** the picture of a fraction of the form $1/b$ or a/b, represent the whole with pictures and/or manipulatives.

We have found that unpacking a Standard in this way is beneficial for two reasons. First, the process offers us a clearer picture of the Standard's mathematical expectations, thus clarifying the end goal of instruction. Second, it affords teachers the opportunity to think deeply about the mathematics, consequently allowing us to identify the differences between CCSSM and the objectives that we have previously taught.

Supporting students in meeting a Standard

With the Standard "unpacked," it becomes clear that meeting the expectations of the Standard will not be accomplished in a single lesson; a unit of instruction that addresses a set of connected Standards is necessary. For example, in grade 3, a unit of instruction might address Fraction Standards 1 and 2, Data 4, and Geometry 2—all of which deal with understanding fractions as quantities and representing them in different ways. Therefore, attention must be given to the instructional decisions regarding the development of a unit that will support students in meeting the Standard. These decisions range from task selection to assessment to the sequence of instruction. Providing a detailed description of this decision-making process and the resulting unit is beyond the scope of this article. However, by considering how to launch our unit for Standard 3.NF.1, we can describe key aspects to the process we identified.

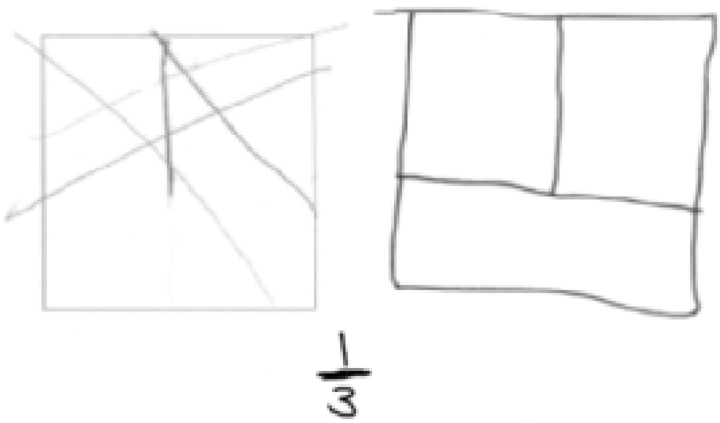
Selecting tasks

In planning a unit, tasks are needed that possess two primary characteristics. First, tasks must hold the potential for engaging students in problem solving. Problem solving gives students the opportunity to connect what they are learning to familiar contexts and ideas. This, in turn, supports the development of understanding (Lambdin 2003). Second, tasks should support students in developing the understanding, knowledge, and skills previously identified. In planning such tasks, we are teaching with the end goal in mind. To begin our unit for Standard 3.NF.1, we identified the Pizza task (Magone, Moskal, and Lane 2002; see **fig. 2**). Our thought process behind selecting this task included the following:

- **The task** involves partitioning a figure into equal parts, a skill developed in second grade. Therefore, students should have access to the mathematics in the task.
- **Because** prerequisite knowledge is involved, the task serves as an informal assessment of this knowledge.
- **A discussion** of the task can easily focus on the mathematical ideas related to or contained within the Standard.
- **Students** should understand the context of equally sharing a pizza, further supporting their ability to access the task.

FIGURE 3

Zach successfully divides the figure into three parts. However, he fails to make the parts of equal size.



Implementing tasks

What does task implementation look like in the classroom? We share a vignette that is intended to help you think about what CCSSM looks like in the classroom (see the **sidebar** on **p. 501**). As the vignette opens, the teacher aims to support students in understanding the task and its associated expectations. Next, the students work on the task with limited interaction with the teacher. Finally, the students have an opportunity to analyze one another's work. Furthermore, the teacher's questions focus students' attention on the mathematics at hand.

Using assessment information

Starting the instructional unit with a task of this nature provides assessment information that can guide instructional decisions. In this section, we examine five students' responses to a task (see **fig. 2**) and consider how the information revealed in their work informs instruction. Student responses are categorized according to whether they are correct or incorrect.

Incorrect or partial responses

Carlos and Zach provided incorrect responses. Carlos divided the square into four pieces (see **fig. 2**). He and many of his classmates were compelled to partition the shape by drawing segments intersecting at the center of the square. Unlike Carlos, Zach successfully divided the figure into three parts (see **fig. 3**). He failed, however, to make the parts of equal

FIGURE 4

If students were to immediately produce this anticipated solution to the task, the task likely failed to engage them in problem solving.

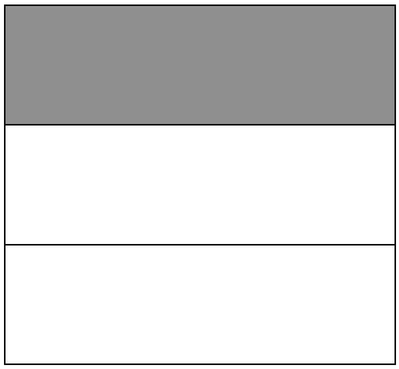
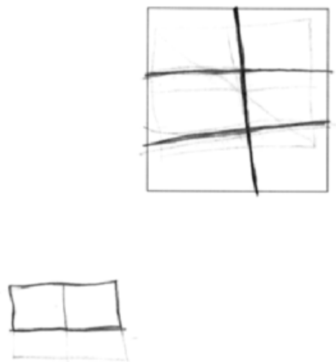


FIGURE 5

Troy's representation partitions the square into three horizontal sections and includes a partition down the middle to produce six equal-size parts. Elena's share is clearly two of those parts.



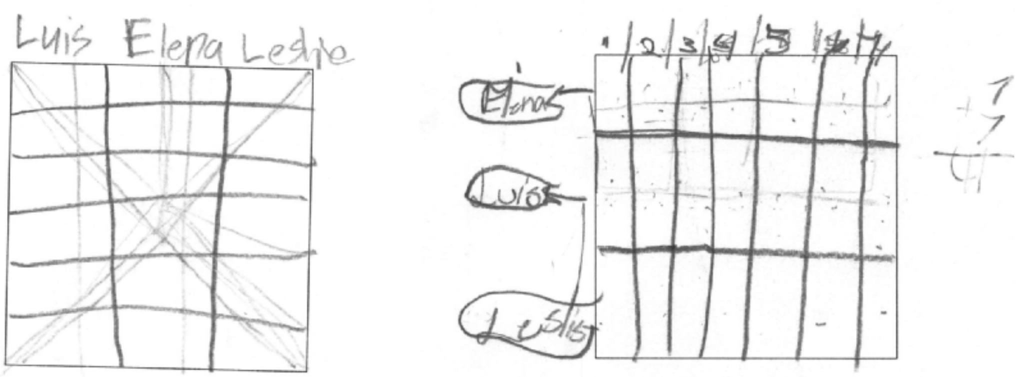
size. Collectively, student responses like these reveal that the students do not possess the prerequisite knowledge needed to begin the unit. The discussion of this task should guide the students toward recognizing the importance of equal-size parts. Additionally, students need opportunities to develop a variety of strategies for partitioning shapes.

Correct responses

None of the students in this class presented the anticipated solution (see **fig. 4**). Troy, however, came the closest. His partitioning (see **fig. 5**) has three horizontal sections, but it also includes a partition down the middle to produce six equal-size parts. In his response to the question, though, he clearly indicates that Elena's share is

FIGURE 6

Amelia and Tamika both divide the square into three columns or rows but add more partitioning, assigning Elena one-third of the pizza. Amelia's representation indicates that she developed an alternative strategy when she recognized that drawing segments intersecting at the center would not work.



two of those parts. Similarly, Amelia and Tamika (see **fig. 6**) divided the square into either three columns or three rows with additional partitioning to produce more parts. Their labeling clearly shows that they have assigned Elena one-third of the pizza. Notice from Amelia's picture (on the left) that she began by attempting to draw segments intersecting at the center. Recognizing that this would not work, however, she developed an alternative strategy.

Finally, consider Enron's work (see **fig. 7**). Looking at his picture alone might lead one to believe that Enron is incorrect. But consider the directions and his fraction. Students are to show how much pizza Elena will get. Perhaps Enron has literally shown only the portion that Elena will receive. If this is the case, then Enron would most likely extend his segments and Elena would indeed receive four-twelfths of the pizza.

We found it intriguing that all the correct responses involved partitioning the pizza into more than three parts, which made us wonder if the problem context of sharing a pizza caused this to happen. After all, how often is a pizza divided in such a way that one part represents one person's share? Regardless, two things are clear from these correct responses: (1) This task engaged students in problem solving, evidenced by the creative responses along with the unsuccessful attempts that were erased, and (2) these students have furnished evidence that they possess the prerequisite skills to begin the unit. Discussing this problem will nicely lead into thinking about the symbolic representation of the fractions.

What if …

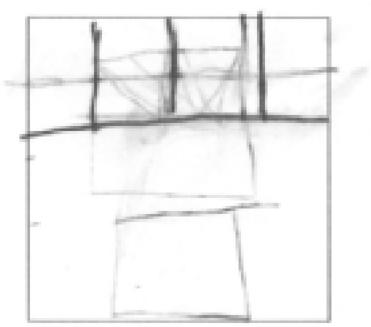
Beyond the correct and incorrect responses that were revealed in our class, consider a different scenario. What if this task were given to students and in a matter of moments they were to produce the anticipated solution (see **fig. 4**)? It would most likely indicate that the students possess the prerequisite skills but the task failed to truly engage them in problem solving. As a result, the students would be ready for the task in **figure 8**, which also supports the development

FIGURE 7

Looking at Enron's picture could lead to the conclusion that he is incorrect. But considering the directions to show how much pizza Elena will get, as well as Enron's fraction, perhaps he is literally showing only the portion that Elena will receive.

Luis, Elena, and Leslie plan to share 1 large, square pizza. Each person will get an equal amount.

1. Show on the picture how much pizza Elena will get.



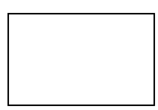
2. What fraction of the pizza will Elena get?

$\frac{4}{12}$

FIGURE 8

If the task in **figure 4** is not enough of a challenge, a task involving fractions also supports the development of understanding, knowledge, and skills from CCSSM 3.NF.1 and is more likely to engage students in problem solving.

Ashley, Kayla, and Tyra equally shared a pizza. Tyra's piece of the pizza is pictured below. Draw a picture of the whole pizza.



What fraction of the pizza did Tyra have?

of understanding, knowledge, and skills from the Standard but is more likely to engage such students in problem solving.

Linking to the Standards

Teaching with the end goal in mind is not limited to mathematical content; one must aim to meet the Standards for Mathematical Practice (SMP). SMP "describes ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter" (CCSSI 2010, p. 8). Task implementation is the key to supporting students in meeting the expectations of SMP. The vignette reveals three keys to implementation that align with SMP.

1. Engaging students in problem solving
   By allowing students to think on their own and creatively tackle the problem, you engage them in problem solving. The teacher guides students in understanding the problem without giving away how to solve it. This, in turn, supports students in recognizing the importance of understanding problems and persevering when solving them (SMP1). By asking students to share their ideas about the problem, the teacher in the vignette supports all students in understanding the problem. The problem also supports students in reasoning about the quantities and relating them to problem contexts (SMP2).

2. Examining student-generated solutions
   After students work the problem, they have an opportunity to share their work. As one student shares, other students should analyze the work, making connections to other solutions and solution processes as well as analyzing the correctness of the solution. The teacher's questions in the vignette, such as Q2, Q4, and Q5, support this analysis of work. In addition, students who are sharing their work should be expected to justify their reasoning (SMP3). Q1 and Q3 from the vignette encourage students to do so.

3. Focusing the discussion on the math
   Through the teacher's questions, students will be forced to communicate clearly (SMP6) as they develop arguments that support their reasoning (SMP3). The teacher's questions also facilitate students' recognition of the patterns and regularities that exist in mathematics (SMP7 and SMP8). Questions such as Q6 from the vignette offer opportunities to express possible generalizations.

Differences between CCSSM content and state frameworks

For grades 3 and 4, the five domains in CCSSM include Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, Measurement and Data, and Geometry. The domain names may call to mind the NCTM Content Strands (NCTM 2000), but differences exist between the two:

1. The Number and Operations strand has been split into two domains: Base Ten and Fractions.
2. The Algebra strand has been combined with Operations.
3. The content for Measurement and Data has been combined into one domain.

CCSSM is an attempt to write Standards to represent "big mathematical ideas." The content requires teachers to think both differently and deeply about the mathematics. The expectations require a deeper understanding of the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mathematics on the students' part. As demonstrated in this article, we have found that unpacking the Standard allows us to effectively address these differences.

Conclusion

CCSSM is different from the objectives that many of us have previously experienced in our state frameworks. Although the mathematical topics of the two may be the same, the mathematical expectations within the Standards require a deeper understanding by teachers and students. In this article, we have demonstrated how using the ideas of *Understanding by Design* (Wiggins and McTighe 2005) can support third- and fourth-grade teachers in "unpacking" a Standard and, in turn, clarify the expectations of a Standard and the end goal of instruction. Supporting students in meeting the expectations of both the Mathematical Practices and Content Standards also includes task selection, task implementation, and instructional decisions based on informal assessments. By unpacking Standard 3.NF.1 and examining a sample task, we have proposed a means for you to think about these processes and navigate through the CCSSM document. With the support of your colleagues and the insights shared here, you too can better understand and implement the Common Core.

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